Information Systems and Retractions: an Elementary Treatment

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ABSTRACT. We show that information systems allow to treat retractions of domains for denotational semantics in more simple and intuitive ways. We consider both algebraic and continuous cases. The basic intuition is that an approximable mapping r describing a retraction $|r|$ should be treated as a candidate to become the entailment relation $\vdash^*$ in the information system to describe $\mathrm{Fix}(|r|)$ as a domain.

In the algebraic case this means that closure operations are the best since only they possess the *reflexivity* property: $\forall u \in Con.\ uru$. For all other retractions we have to exclude assertions that become non-reflexive, which means that we are generally unable to describe their sets of fixed points by information systems. Yet the situation can be somewhat salvaged if r performes all its entailments *via* reflexive elements. Formally, we can write it as $\forall u, w \in Con.\ urw \Rightarrow \exists v \in Con.\ urv, vrv, vrw$. v here is an element retaining reflexivity. This formula is the necessary and sufficient condition for the retraction $|r|$ to be *finitary*, because we can use elements retaining reflexivity as a backbone to build an information system that would describe a domain isomorphic to $\mathrm{Fix}(|r|)$ as an algebraic cpo. Since we still have to exclude non-reflexive elements, this isomorphism cannot be refined to equality. This criterion of finitarity can be rewritten as $\forall u, w \in Con.\ urw \Rightarrow \exists v \in Con.\ urv, vrv, v \vdash^* w$ for retractions and as $\forall u, w \in Con.\ urw \Rightarrow \exists v \in Con.\ u \vdash^* v, vrv, v \vdash^* w$ for projections.

Continuous information systems replace reflexivity $A \vdash A$ by a weaker property of *inverse modus ponens*: $A \vdash C \Rightarrow \exists B.\ A \vdash B, B \vdash C$ [Hoofman 93]. This totally eliminates the assymetry of the algebraic case. All retractions are equivally good to serve as entailment relations in information systems describing their respective $\mathrm{Fix}(|r|)$'s.

We discuss constructions of information systems to describe $\mathrm{Fix}(|r|)$ exactly or up to isomorphism, constructions of domains of finitary projections and retractions, and other related matters.

0. INTRODUCTION. In [Scott 82] Dana Scott introduced *information systems* to describe algebraic domains for denotational semantics. An information system describing a domain is a logical calculus, and the elements of the domain are *theories* in this calculus — consistent, deductively closed sets of propositions.

[Hoofman 93] generalized information systems to the case of continuous domains. The price for this generalization is the loss of the ordinary reflexivity rule $A \vdash A$, and its replacement by the (slightly!) weaker rule of *inverse modus ponens*: $A \vdash C \Rightarrow \exists B.\ A \vdash B, B \vdash C$. The notion of a theory, i.e. a domain element, is slightly modified as well. In the spirit of the *inverse modus ponens* rule every statement of the theory must follow from an "axiom" — another statement — belonging to the theory. Of course, ordinary requirements of consistency and deductive closeness remain.

This generalization, together with the earlier efforts by V.Yu.Sazonov to describe continuous domains via information systems allowing for infinite logic, demonstrates the value and the place of algebraic domains. It is possible to describe non-algebraic domains by logical calculi, but unusual logic is required as a payment for non-algebraicity.

This makes it interesting to study *finitary retractions*, i.e. retractions with algebraic sets of fixed points, and their properties. Many people seem to study finitary retractions, but many important and fundamental results seem to remain unpublished. The use of information systems greatly simplifies studies of finitarity, yet I was not able to find any applications of information systems to finitarity in print.

I asked a number of experts about availability of these results. Certain related material is contained in [Amadio, Bruce, Longo 86; Barendregt 84; Berardi 91; Huth 91], but none of these papers contain materials presented here, except where explictly noted. Some experts recalled seeing some of the results presented here in unpublished manuscripts or PhD thesises, all of which are difficult to obtain. Among relevant manuscripts we should name [McCracken 84]. These results, while being basic and fundamental, are not overly difficult to obtain for a competent researcher. Because they are not properly published some of them are being independently rediscovered by a number of people, while remaining virtually unknown to the more wide research community. I hope that in the remaining four months I will be able to acknowledge contributions to this subject more completely and to refer to the efforts of those who codiscovered the results presented here but never published them.

Why is it of interest to have these results widely available? Dana Scott motivated introduction of information systems by the necessety to make the theory of domains more intuitive and understandable for its users, and the need was indeed present. Yet approximately half of the researchers in the domain theory admit that they do not feel comfortable with information systems and their applications. On the other hand many potential users outside the field, including some distinguished researchers in such closely related field as λ-calculus, state that the level of abstraction in the domain theory does not pay off and they are not seeing recent applications significant enough to justify the effort.

Properties of retraction were always in the basis of the theory, yet they are often considered difficult and counterintuitive. Indeed, it is likely that the most of those who learn or (re)discover at the first time that non-finitary projections exist, react with surprise and disbelief. Algebraic information systems allow to make this treatment elementary — it is not necessary to use topology, categories, or complicated lattice theory. Not only the treatment becomes elementary, but it is also easier — resulting formulas are actually much simpler. Continuous information systems allow us to enhance further the understanding of both finitary and non-finitary retractions.

Sections 1 and 2 below are introductory. Section 3 treats the finitarity of projections. Criteria are discussed and a simple example of a non-finitary projection is given. Domains of fixed points of finitary projections are constructed and discussed, including the explanation why they are not as good as fixed points of closure operations. Inverse limits are discussed. The domain of finitary projections is constructed.

Section 4 deals with the finitarity of retractions. Again we discuss criteria, build domains of fixed points, and talk about the domain of finitary retractions. Then we discuss two potentially interesting subclasses of finitary retractions — quasi-minimal retractions and strongly finitary retractions.

Sections 5 and 6 deal with continuous information systems and domains.

1. ALGEBRAIC INFORMATION SYSTEMS AND APPROXIMABLE MAPPINGS, ALGEBRAIC DOMAINS AND CONTINUOUS FUNCTIONS. The full paper will contain a brief tutorial. The appropriate references are [Scott 82; Larsen, Winskel 84; Hoofman 93]. Our notation essentially follows [Larsen, Winskel 84]. We distinguish between $\vdash \subseteq Con \times D$ and $\vdash^* \subseteq Con \times Con$.

Meanwhile this material is a prerequisite for the readers of this extended abstract.

2. RETRACTIONS, PROJECTIONS AND FINITARITY. A continuous function $|r| : |A| \to |A|$ is a retraction if $|r| = |r| \circ |r|$. A retraction $|r|$ is a projection if $|r| \sqsubseteq id$. A retraction $|r|$ is a closure operation if $|r| \sqsupseteq id$.

An approximable mapping r defines a retraction iff $\forall u, w \in Con.\ urw \Leftrightarrow \exists v \in Con.\ urv, vrw$. An approximable mapping r defines a projection iff $\forall u, w \in Con.\ urw \Rightarrow u \vdash^* w$ and $\exists v \in Con.\ urv, vrw$. An approximale mapping r defines a closure operation iff $\forall u \in Con.\ uru$ and $\forall u, v, w \in Con.\ urv, vrw \Rightarrow urw$.

A retraction is *finitary* if the set of its fixed points is algebraic. Equivalently, a retraction $|r| : |A| \to |A|$ is finitary if there is a domain $|B|$ and a pair of continuous functions $|i| : |B| \to |A|$ and $|j| : |A| \to |B|$, such that $|j| \circ |i| = id_B$ (we say, that $|i|, |j|$ form an *embedding-retraction pair* performing a *retraction of* $|A|$ *onto* $|B|$) and $|r| = |i| \circ |j|$.

Projections are called finitary if they are finitary retractions. All closure operations are finitary retractions.

3. FINITARY PROJECTIONS VS. CLOSURES.

Finitary projections. First consider finitary projections for domains taken as abstract cpo's. A projection $|p| : |A| \to |A|$ is finitary iff $\forall x \in |A|.\ x = |p|(x) \Rightarrow x = \sqcup\{x_0 \in |A|_0 \mid x_0 = |p|(x_0), x_0 \sqsubseteq x\}$. This criterion can be obtained from [Huth 91, Lemma 1(ii)], which in effect says that the set of finite elements in Fix($|p|$) is $\{x_0 \in |A|_0 \mid x_0 = |p|(x_0)\}$.

Consideration of information systems easily produces a much nicer criterion: a continuous function $|p| : |A| \to |A|$ is a finitary projection iff $\forall u, w \in Con_A.\ upw \Rightarrow \exists v \in Con_A.\ u \vdash^*_A v, vpv, v \vdash^*_A w$. It does not require consideration of non-finite elements and allows to select finitary projections from the space of all continuous functions rather than from the space of projections. It can be literally rewritten in term of abstract cpo's: $\forall x_0, z_0 \in A_0.\ z_0 \sqsubseteq |p|(x_0) \Rightarrow \exists y_0 \in A_0.\ x_0 \sqsupseteq y_0 \sqsupseteq z_0, y_0 = |p|(y_0)$. Yet, one is unlikely to chose this criterion if he does not consider information systems because it involves inequality $z_0 \sqsubseteq |p|(x_0)$. Also notice that in this case we have to consider $|p|(x_0)$ which is generally not finite.

A non-finitary projection. This counterexample goes along the line of Ershov's examples for $P(\omega)$ described in [Berardi 91, p.213], but it is very simple, because the domain under consideration does not have incomparable elements. Our domain will be as close to the vertical line of real numbers as it is possible for an algebraic cpo. Define D_R = {real nonnegative numbers}, Con_R = {all finite subsets of D_R}, $\{d_1, \ldots, d_n\} \vdash_R d$ iff

$d \leq max\{d_1, \ldots, d_n\}$. Then $\bot_R = \{0\}, \top_R = D_R$, and between them we have two representatives for each strictly positive real number: a finite element $|r|_0 = \{d \in D_R \mid d \leq r\}$ is a finite element corresponding to r, and right *under* it we have a non-finite element corresponding to r, specifically $|r| = \{d \in D_R \mid d < r\}$. One may slightly modify this construction by taking any dense set of real non-negative numbers containing 0 as D_R, e.g. rational numbers. This will reduce the number of finite elements, but we are still going to have a non-finite element for each real number, and the subsequent construction will go through. Our choice of D_R is determined mainly by the symmetry reasons.

The non-finitary projection is defined by $d_1 p d_2$ iff $d_2 < d_1$ (the extension onto Con_R is obvious). Then $\bot_R, \top_R$ and all non-finite elements $|r|$ corresponding to positive numbers are fixed points of $|p|$, but all finite elements corresponding to positive numbers move down: $|p|(|r|_0) = |r|$. The resulting set of fixed points does not have any finite elements except $\bot_R$, and, hence, is not algebraic.

$|p|$ is a *non-extensive* projection [Berardi 91], i.e. $x = |p|(x)$ and finiteness of x together imply $x = \bot$. This example was communicated to me by Alexander Saevsky.

Domains of fixed points of finitary projections vs. domains of fixed points of closure operations. It is shown [Scott 76, Theorem 5.1; Bukatin 94, Theorem 2.5 and corollaries] that all closure operations are finitary retractions, and that domains of their fixed points are subsets of the original domains. On the contrary and somewhat surprisingly, fixed points of finitary projections do not literally form domains determined by information systems. Instead, given a finitary projection $|p|$ one can construct an information system P such that $|P|$ and Fix($|p|$) are isomorphic as algebraic cpo's. Let us look at the details.

Define $P = (D_P, Con_P, \vdash_P)$ as follows:

$$D_P = \{u \in Con_A \mid upu\};$$

$$Con_P = \{\{u_1, ..., u_n\} \subseteq_{Fin} D_P \mid u_1 \cup \ldots \cup u_n \in Con_A\};$$

$$\{u_1, ..., u_n\} \vdash_P u \text{ iff } u_1 \cup \ldots \cup u_n \vdash^*_A u.$$

The construction of P includes two features — conjunctive completion and elimination of assertions, such that $d \not p\, d$. Both change Fix($|p|$) and are responsible for the fact that we have only $|P| \cong$ Fix($|p|$), but not $|P| =$ Fix($|p|$).

The first step — conjunctive completion — is not so essential and can be avoided if we start with a conjunctively complete algebraic information system A from the very beginning (A is conjunctively complete if $\forall u \in Con_A.\exists d \in D_A.u \vdash_A d, \{d\} \vdash_A^* u$). The role of conjunctive completeness can be illustrated by the following example. Consider the domain $|D| = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$. To differentiate between the projection onto $\{\emptyset, \{1,2\}\}$ and the identity projection, we need an element ∇, such that $\{\nabla\} \vdash^* \{1,2\}, \{1,2\} \vdash \nabla$. On the other hand, if we consider $|D'| = \{\emptyset, \{1\}, \{2\}, \{1,2,\nabla\}\}$, conjuctive completion would not be necessary.

Assuming the property of conjunctive completeness we can modify the construction of P so, that $D_P = \{d \in D_A \mid dpd\}$, $\forall u \subseteq D_P.\ u \in Con_P$ iff $u \in Con_A$, $\forall u \in Con_P, d \in D_P.\ u \vdash_P d$ iff $u \vdash_A d$. The elimination of assertions d, such that $d \not p d$, unfortunately remains essential, and we cannot get rid of it. To illustrate this, consider conjunctively complete $|D| = \{\emptyset, \{1\}, \{1,2\}, \{1,3\}\}$ and its projection onto $\{\emptyset, \{1,2\}, \{1,3\}\}$. The corresponding $|P|$ is equal to $\{\emptyset, \{2\}, \{3\}\}$, which is *isomorphic and informationally equivalent but not equal* to $\{\emptyset, \{1,2\}, \{1,3\}\}$. Note, that $\{\emptyset, \{1,2\}, \{1,3\}\}$ cannot be described by a closure operation on $|D_\top|$. In fact, $\{\emptyset, \{1,2\}, \{1,3\}\}$ cannot be obtained as domain from any algebraic information system (consider, what would be $\overline{\{1\}}$ to see that), although it is isomorphic to such a domain. We cannot thus obtain $|P| = \text{Fix}(|p|)$ and have to live with $|P| \cong \text{Fix}(|p|)$.

[Bukatin 94] agrues that these considerations imply that only sets of fixed points of closure operations (actually, closure operations on $A_\top$) can be considered true subdomains in the algebraic case, while sets of fixed points of projections and arbitrary retractions cannot even when these projections and retractions are finitary.

In Section 6 we will consider retractions for continuous information systems and will show that in the continuous case the assymmetry between closure operations and other retractions disappears. The elimination of the reflexivity requirement from the underlying logic will play the key role.

Also notice that our construction of P can be applied even to non-finitary projections. In this case elements $x = |p|(x)$ not expessible as $\sqcup\{x_0 \in |A|_0 \mid x_0 = p(x_0), x_0 \sqsubseteq x\}$ will be lost. In particular, non-extensive projections would yield a one-element domain.

Inverse limits. In the full paper we discuss the construction of the inverse limits for algebraic information systems. [Larsen, Winskel 84] deals in details with the partial case of sequences of embedding-projection pairs, where $A \trianglelefteq B$ means $D_A \subseteq D_B$, and Con_A and $\vdash_A$ are obtained as restrictions of Con_B and $\vdash_B$ onto D_A. We will explain why this partial case is likely to be sufficient to cover all relevant embedding-projection sequences.

The domain of finitary projections. The set of all finitary projections $|A| \to |A|$ can be obtained as the set of fixed points of the finitary projection $|Pr|$ of $|A \to A|$. Specifically, if $g \in |A \to A|$ define $f = |Pr|(g)$ by ufw iff $\exists v \in Con_A.\ u \vdash_A^* v, vgv, v \vdash_A^* w$.

It is easy to check that f is a finitary projection, and that if g is a finitary projection then $g = |Pr|(g)$. To prove that $|Pr|$ itself is a continuous function and a finitary projection, one should notice that $\{(u_1, w_1), \ldots, (u_n, w_n)\}$ $Pr\{(u, w)\}$ iff $\exists v.\ u \vdash_A^* v, v \vdash_A^* w, \{(u_1, w_1), \ldots, (u_n, w_n)\} \vdash_{A \to A} (v, v)$.

4. FINITARY RETRACTIONS. It is helpful to consider finitary retractions as a generalization of both finitary projections and closure operations. Closure operations give us a lot of examples where non-finite elements of the original domain serve as finite elements of the set of fixed points. But all these elements are images of the finite elements of the original domain. These considerations allow us to write the following criterion: a retraction $|r| : |A| \to |A|$ is finitary iff $\forall x \in |A|.\ x = |r|(x) \Rightarrow x = \sqcup\{|r|(x_0) \mid x_0 \in |A|_0, x_0 \sqsubseteq |r|(x_0) \sqsubseteq x\}$.

Again this looks much easier in terms of information systems and approximable mappings. A retraction $|r|$ is finitary iff $urw \Rightarrow \exists v \in Con_A.\ urv, vrv, vrw$. One can easily rewrite this as $|r|$ is finitary iff $urw \Rightarrow \exists v \in Con_A.\ urv, vrv, v \vdash_A^* w$. Notice that vrv above shows that a retraction $|r|$ is finitary if and only if the associated approximable mapping r entails w from u via some v for which reflexivity of this new entailments holds (we appeal here to the intuition that approximable mappings are input-output entailment relations, and when the corresponding continuous functions are retractions the resemblance to the standard entailment increases).

Domains of fixed points of finitary retractions. [Bukatin 94] shows that to consider all subdomains of a domain $|A|$ one should create the domain $|A_\top|$ and consider closure operations on this domain ($|r|(\bot) = \top$ is forbidden). After that to obtain a subdomain one should exclude the top element from the resulting set of fixed points. Because finitary retractions are generalizations of finitary projections and closure operations we must do the same thing for them to treat this matter in the full generality.

Specifically, consider $|A_\top|$, $\top = \overline{\{\nabla\}}$, where ∇ is an extra element added to D_A that entails everything and is entailed by previously inconsistent conjuctions. Consider a finitary retraction $|r| : |A_\top| \to |A_\top|$, such that

$|r|(\perp) \neq \top$. Then construct the information system R uniting appropriate constructions for closure operations and finitary projections:

$D_R = \{u \in Con_{A_\top} \mid uru, u \not\vdash \{\nabla\}\}$;

$\{u_1, \ldots, u_n\} \in Con_R$ iff UrU and $U \not\vdash \{\nabla\}$, where $U = u_1 \cup \ldots \cup u_n$;

$\{u_1, \ldots, u_n\} \vdash_R u$ iff $(u_1 \cup \ldots \cup u_n)\ r\ u$.

It is not difficult to prove that $|R| \cong \mathrm{Fix}(|r|) \setminus \{\top\}$.

Domain of finitary retractions. It is possible to obtain a finitary retraction of an appropriately modified functional space, such that its fixed points are exactly finitary retractions. More specifically, we have to mate the construction of the domain of finitary projections via a finitary projection of $|A \to A|$ (obtained above), and the construction of the domain of closure operations $|r|$, $|r|(\perp) \neq \top$ on $|A_\top|$ via a closure operation on $|(A_\top \to A_\top)_\top|$ [Bukatin 84].

As a result, we can construct a finitary retraction of $|(A_\top \to A_\top)_\top|$, such that its fixed points (excluding the top one) are exactly finitary retractions $|r|$ of $|A_\top|$, such that $|r|(\perp) \neq \top$. We do not have space here for detailed treatment.

Special cases of finitary retractions. Here we look at two special subclasses of the class of finitary retractions, which may deserve special attention.

Let us look at the criteria for finitarity of retractions and projections once again. For retractions it is $urw \Rightarrow \exists v.\ urv, vrv, vrw$, which can be rewritten as $urw \Rightarrow \exists v.\ urv, vrv, v \vdash^* w$. For projections it is $upw \Rightarrow \exists v.\ u \vdash^* v, vrv, v \vdash^* w$. There is an intermediate case — retractions satisfying the property that $urw \Rightarrow \exists v.\ u \vdash^* v, vrv, vrw$. This class of retractions includes both finitary projections and closure operations, and the retractions from this class possess a certain weak property of minimality, hence they can be called *quasi-minimal*.

Another class is obtained from the observation that while for finitary projections finite elements of sets of fixed points are finite in the original domain, it is not generally so for finitary retractions. We can call the retractions possessing this property *strongly finitary*. The class of strongly finitary retractions includes the class of finitary projections, but does not include the class of closure operations.

Both quasi-minimal retractions and strongly finitary retractions may be worth more detailed studies.

5. NON-REFLEXIVE LOGIC OF CONTINUOUS INFORMATION SYSTEMS. The continuous information systems were introduced in [Hoofman 93] to generalize the information systems approach to the case of continuous Scott domains. In the underlying logic the reflexivity rule, $A \vdash A$, is replaced by the *inverse modus ponens* rule, $A \vdash C \Rightarrow \exists B.\ A \vdash B, B \vdash C$, which is actually only slightly weaker.

This leads to the following modifications in the definitions. In the definition of an information system reflexivity ($d \in u \Rightarrow u \vdash d$) goes, and the transitivity rule now has $\Leftrightarrow$ instead of $\Rightarrow$ ($\exists v \in Con.(\forall d \in v.(u \vdash d)$ and $v \vdash d_0) \Leftrightarrow u \vdash d_0$), thus including both *modus ponens* and *inverse modus ponens*. Also non-reflexivity forces us to state separately trasitivity with inclusion ($u \subseteq v, u \vdash d \Rightarrow v \vdash d$), and the rule that entailment preserves non-contradiction has to be modified.

Elements are now not just consistent, deductively closed sets of assertions, i.e. usual theories, but we require that every assertion follow from something else in the theory (was trivially true in the reflexive case).

Continuous functions are defined by continuous approximable mappings. Here the usual transitivity with $\vdash_A^*$ and $\vdash_B^*$ ($u' \vdash_A^* u, ufv, v \vdash_B^* v' \Rightarrow u'fv'$) is replaced by the transitivity in both directions ($(\exists u \in Con_A, v \in ConB.\ u' \vdash_A^* u, ufv, v \vdash_B^* v') \Leftrightarrow u'fv'$) and the special transitivity with inclusion ($u \subseteq v, ufw \Rightarrow vfw$).

6. RETRACTIONS AND CONTINUOUS INFORMATION SYSTEMS. Consider a continuous information system A and a continuous approximable mapping r corresponding to the retraction $|r| : |A| \to |A|$. One can prove that if we replace $\vdash_A$ by $\vdash_R$, where $u \vdash_R d$ iff $ur\{d\}$, then we will obtain a continuous information system R and $|R| = \mathrm{Fix}(|r|)$.

Of course, if one still wants to have the ability to obtain, say $\{\emptyset, \{1\}, \{2\}\}$ from $\{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$, one has to consider retractions of $|A_\top|$ and to exclude $\top$ from the sets of fixed points along the lines of [Bukatin 94].

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